

1. A “non-Newtonian fluid” is a fluid for which the viscosity depends on how the fluid is flowing, rather than being constant. For example, a mixture of corn starch and water behaves like a normal liquid if you run your finger through it slowly, but it stiffens up dramatically if you try to run your finger through it quickly. Under the right conditions, an object moving through such a fluid might experience a drag force proportional to the *cube* of the velocity, rather than the linear or quadratic drag we talked about in class.

a) Suppose that a cart with mass m is moving in a straight line on a horizontal track while submerged in a non-Newtonian fluid which applies a drag force with magnitude cv^3 , where v is the cart’s velocity. At time $t=0$, the cart is at position $x=0$ and has initial velocity v_0 . Solve the equation of motion to find $v(t)$, the velocity as a function of time. In particular, show that it can be written in the form

$$v(t) = \frac{v_0}{\sqrt{1 + t/\tau}} \quad . \quad (10 \text{ points})$$

b) What is τ in terms of the physical parameters in the problem? What are the units of τ ? (5 points)

c) At what time will the cart’s speed have dropped to exactly half of its initial value? Express your answer using τ . (5 points)

d) Find the position as a function of time $x(t)$ for the cart. (5 points)

e) Approximate $x(t)$ for small t by using the binomial expansion to second order in t/τ . Again, express your answer using τ . [See the formula sheet for the binomial expansion.] (5 points)

2. A certain one-dimensional system has a potential energy function $U(x) = C \left(\sqrt{x} + \frac{L}{\sqrt{x}} \right)$, where C and L are constants.

a) Sketch the potential $U(x)$ and find the angular frequency ω_0 of small oscillations around the minimum of this potential for an object of mass m . (10 points)

b) Now suppose a damping force is added which is given by $F_d = -b\dot{x}$ with b equal to $(m\omega_0/5)$. Does that make the frequency of the oscillations increase or decrease? What is the new oscillation frequency? (5 points)

c) The system is now driven with a sinusoidal force, $F(t) = F_0 \cos(\omega t)$. Consider three different driving frequencies: $\omega = \omega_0/5$, ω near resonance, and $\omega = 5\omega_0$. For each one, describe the phase of the motion $x(t)$ relative to the phase of the driving force. (e.g., does the motion lead or lag the driving force in each case? If so, by how much? Give semi-quantitative answers and sketches.) (5 points)

3. Consider forces as a function of position (x,y,z) .

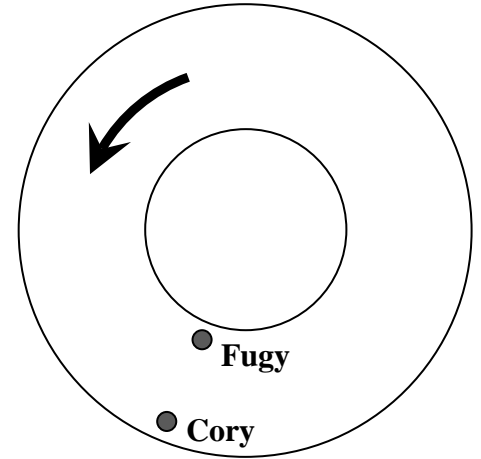
a) One of the following forces is conservative, and the other is not. Show which is which. (10 points)

$$\text{A: } \vec{F}(x, y, z) = (x^2) \hat{x} + (x^2 + 2yz) \hat{y} + (2xz) \hat{z}$$

$$\text{B: } \vec{F}(x, y, z) = (2xy) \hat{x} + (x^2 + 2yz) \hat{y} + (y^2) \hat{z}$$

b) For the force from part (a) which is conservative, determine its potential energy $U(x,y,z)$ where $U(0,0,0) = P_0$ (some constant). (10 points)

4. Two children Cory and Fugy, like to ride the carousel at the local park. Suppose that Cory is riding in the outer ring of horses, at a radius r_2 from the carousel's axis, while Fugy is riding in the inner ring, r_1 from the carousel's axis, directly inward radially from Cory. The carousel rotates with a period T in the counter-clockwise direction when viewed from above.



a) Suppose that Cory and Fugy are each holding a tennis ball (mass m) while the carousel rotates. In their (rotating) reference frame, what are the magnitude and direction of the centrifugal force on each ball? What fraction of the ball's true weight are each of those? (10 points)

b) If Cory opens his hand to let his ball drop, describe its motion (semi-quantitatively) as seen in his reference frame, and also as seen in the reference frame of someone standing beside the carousel. (Neglect air resistance.) (10 points)

c) Fugy throws her ball directly in Cory's direction (i.e. radially outward) with a speed of v_0 . What is the magnitude of the sideways force on the ball in her reference frame? When the ball reaches Cory, will it be in front of or behind the point she was aiming for? (5 points)

d) When the ball reaches Cory, estimate quantitatively (in terms of the quantities given above) how far it will be in front of or behind the spot where Fugy was aiming. (5 points)

